STRUCTURAL DESIGN CHEATSHEET

A summary of the most important formulas, equations & diagrams

\[ M = \frac{ql^2}{8} \]
This Structural Design Cheatsheet covers the engineering formulas and equations I use regularly as a structural engineer. It’s a “short” summary of many of the blog posts we published on structuralbasics.com.

While this document covers many important formulas, be aware that it doesn’t cover every formula. And as for any structural engineering book or formula sheet, some explanations and formulas are simplified. For example, the wind load is only shown from one direction, or the steel Eurocode provides also formulas for bending and shear, normal force and bending, etc.

Structural engineering is simply too complex to cover every design situation.

Use the content of the document as a guide, but question everything critically.

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With that out of the way, I hope this document helps you get a good overview of structural design.
**Simply supported beam – Line load**

Reactions:
\[ R_a = R_b = \frac{ql}{2} \]

Maximum shear force:
\[ V_a = V_b = \frac{ql}{2} \]

Maximum bending moment:
\[ M_{\text{max}} = \frac{ql^2}{8} \]

**Simply supported beam – Point load**

Reactions:
\[ R_a = Q \frac{b}{l}, R_b = Q \frac{a}{l} \]

Shear forces:
\[ V_a = Q \frac{b}{l}, V_b = Q \frac{a}{l} \]

Maximum bending moment:
\[ M_{\text{max}} = Q \frac{ab}{l} \]

For more moment and shear force formulas of the simply supported beam click [here](#).

**Cantilever beam – Line load**

Reactions:
\[ R_a = q \cdot l, M_a = -\frac{1}{2} \cdot q \cdot l^2 \]

Shear forces:
\[ V_a = -q \cdot l \]

Maximum bending moment:
\[ M_{\text{max}} = -\frac{1}{2} \cdot q \cdot l^2 \]
### Cantilever beam – Point load

![Cantilever Beam Diagram](image)

**Reaction forces:**
- \( R_a = Q \)

**Max. shear forces:**
- \( V_a = -Q \)

**Max. bending moment:**
- \( M_{max} = Q \cdot a \)

For more moment and shear force formulas of the cantilever beam click [here](#).

### 2 span continuous beam – Line load on 2 spans

![Continuous Beam Diagram](image)

**Reaction forces:**
- \( R_a = R_c = \frac{3}{8} q \cdot l, R_b = \frac{5}{4} q \cdot l \)

**Shear forces:**
- \( V_a = V_c = \frac{3}{8} q \cdot l, V_b = \frac{5}{8} q \cdot l \)

**Bending moments:**
- \( M_{max} = \frac{9}{128} \cdot q \cdot l^2, M_b = -\frac{1}{8} \cdot q \cdot l^2 \)

For more moment and shear force formulas of the 2 span continuous beam click [here](#).

### 2 span continuous beam – Line load on 1 span

![Continuous Beam Diagram](image)

**Reaction forces:**
- \( R_a = \frac{7}{16} q \cdot l, R_b = \frac{5}{4} q \cdot l, R_c = -\frac{1}{16} q \cdot l \)

**Shear forces:**
- \( V_a = R_a, V_b = -\frac{9}{16} q \cdot l, V_c = -R_c \)

**Max. bending moment:**
- \( M_{max} = \frac{49}{512} \cdot q \cdot l^2, M_b = -\frac{1}{16} \cdot q \cdot l^2 \)

For more moment and shear force formulas of the 2 span continuous beam click [here](#).
3 span continuous beam – Line load on 3 spans

Reation forces:
\[ R_a = R_d = 0.4 \cdot q \cdot l , \]
\[ R_b = R_c = 1.1 \cdot q \cdot l \]

Shear forces:
\[ V_a = V_d = \pm 0.4 \cdot q \cdot l , \]
\[ V_b(-) = -V_c(+) = -0.6 \cdot q \cdot l , \]
\[ V_b(+) = -V_c(-) = 0.5 \cdot q \cdot l \]

Bending moment:
\[ M_{\text{max}} = 0.08 \cdot q \cdot l^2 , M_b = -0.1 \cdot q \cdot l^2 \]

3 span continuous beam – Line load on 2 spans

Reation forces:
\[ R_a = 0.383 \cdot q \cdot l , R_b = 1.2 \cdot q \cdot l , \]
\[ R_c = 0.45 \cdot q \cdot l , R_d = -0.033 \cdot q \cdot l \]

Shear forces:
\[ V_a = 0.383 \cdot q \cdot l , V_d = 0.033 \cdot q \cdot l \]
\[ V_b(-) = -0.617ql , V_b(+) = -0.583ql \]
\[ V_c(-) = -0.417ql , V_c(+) = 0.033ql \]

Bending moments:
\[ M_{\text{max}} = 0.074ql^2 , M_b = -0.117ql^2 , \]
\[ M_{bc} = 0.053ql^2 , M_c = -0.033ql^2 \]

3 span continuous beam – Line load on 1 span

Reation forces:
\[ R_a = 0.433 \cdot q \cdot l , R_b = 0.65 \cdot q \cdot l , \]
\[ R_c = -0.1 \cdot q \cdot l , R_d = 0.017 \cdot q \cdot l \]

Bending moments:
\[ M_{\text{max}} = 0.094ql^2 , M_b = -0.067ql^2 , \]
\[ M_c = 0.017ql^2 \]

For more moment and shear force formulas of the 3 span continuous beam click here.
**GEOMETRY**

**Centroid**

The centroid is a point of a cross-section that represents the *center of mass*. It’s the point at which the entire area of the section can be assumed to be concentrated.

\[
z = \frac{\sum_{i=1}^{n} A_i \cdot z_i}{A}
\]

With,

- \(n\): number of “parts” of the cross-section (\(n=2\) in the shape above)
- \(A_i\): area of the part \(i\)
- \(z_i\): distance of the centroid of part \(i\) from the point of origin (top fibre in the shape above)
- \(A\): area of the section

**Moment of inertia**

The moment of inertia is a measure of an object’s resistance to changes in rotational motion. It is used to calculate the bending stresses that a structural element will experience when subjected to a load.

\[
I = \sum_{i=1}^{n} I_{0,i} + A_i \cdot z_i^2
\]

With,

- \(n\): number of “parts” of the cross-section (\(n=2\) in the shape above)
- \(I_{0,i}\): moment of inertia of part \(i\)
- \(A_i\): area of the part \(i\)
- \(z_i\): distance of the centroid of part \(i\) from the centroid \(c\)
GEOMETRY

Section modulus

Section modulus of top fibre:
\[ W = \frac{I}{z_1} \]

Section modulus of bottom fibre:
\[ W = \frac{I}{z_2} \]

With,

- \( I \): moment of inertia around strong axis
- \( z \): distance of fibre from centroid

Moment of inertia formulas

Rectangular section

\[ I_y = \frac{wh^3}{12} \]
\[ I_z = \frac{hw^3}{12} \]

I/H section

\[ I_y = \frac{wh^3}{12} - \frac{(w-t_w)(h-2t_w)^3}{12} \]
\[ I_z = \frac{hw^3}{12} - \frac{(w-t_w)^3(h-2t_w)}{12} \]

Hollow circular section

\[ I_y = \frac{(D^4 - d^4) \cdot \pi}{64} \]
\[ I_z = \frac{(D^4 - d^4) \cdot \pi}{64} \]

Rectangular hollow section

\[ I_y = \frac{BH^3 - bh^3}{12} \]
\[ I_z = \frac{HB^3 - hb^3}{12} \]

For more moment of inertia formulas of other sections click [here](#).
Overview of loads used in structural design

The “common” characteristic loads that are used in the structural design of buildings are:
- Dead load
- Live load
- Horizontal wind load on walls
- Wind loads on roofs
- Snow load
- Soil pressure
- Seismic load

Dead load

The dead load represents permanent loads, such as the self-weight of structural and non-structural building materials. The self-weight of a concrete slab, a timber truss roof and windows are examples of the dead load. The weight is calculated and then applied to the structural member that carries it.

Area dead load:

\[ g_k = \text{density of element} \cdot \text{thickness} \left[ \frac{kN}{m} \right] \]

Live load

The live load represents variable loads such as weight of people, furniture, cars, office equipment, etc. that can change over time. It's an approximation for structural engineers to estimate the additional weight (excluding self-weight) that can act on structures due to different room categories.

You’ll find the values of the live load for different room classes in EN 1991-1-1 Table 6.2 and the National Annex of your country.

Click here for the in-depth article.
Horizontal wind loads on walls

The horizontal wind load on buildings are split up in 4 or 5 areas (A, B, C, D, E) with different load values.

Wind loads on walls:

\[ w_k = q_p \cdot c_{pe} \]

With,

- \( q_p \) peak velocity pressure
- \( c_{pe} \) pressure coefficient for each area according to EN 1991-1-4 Table 7.1

Click [here](#) for step-by-step guide for the peak velocity pressure. And [here](#) for the full calculation guide for horizontal wind loads.
Wind loads on pitched roofs

The wind loads on pitched roofs are split up in 4 or 5 areas (F, G, H, I, J) with different load values.

\[ w_k = q_p \cdot c_{pe} \]

With,

- \( q_p \) peak velocity pressure
- \( c_{pe} \) pressure coefficient for each area according to EN 1991-1-4 Table 7.4a.

Click [here](#) for the calculation guide for wind loads on pitched roofs.
Wind loads on flat roofs

The wind loads on flat roofs are split up in 4 areas (F, G, H, I) with different load values.

Wind loads on flat roofs:

\[ w_k = q_p \cdot c_{pe} \]

With,

- \( q_p \): peak velocity pressure
- \( c_{pe} \): pressure coefficient for each area according to EN 1991-1-4 Table 7.2.

Click here for the calculation guide for wind loads on flat roofs.
Snow loads on flat and pitched roofs

The snow load on pitched and flat roofs is calculated as (EN 1991-1-3 (5.1)):

\[ s = \mu_i \cdot C_e \cdot C_t \cdot s_k \]

With,

- \( \mu_i \): snow load shape coefficient
- \( C_e \): exposure coefficient
- \( C_t \): thermal coefficient
- \( s_k \): characteristic snow load value on ground

Click [here](#) for a calculation guide for snow loads on pitched roofs. And [here](#) for flat roofs.
Ultimate limit state (ULS)

\[
\sum_{j \geq 1} \gamma_{G,j} \cdot G_{k,j} + \gamma_{Q.1} \cdot Q_{k.1} + \sum_{i > 1} \gamma_{Q,i} \cdot \psi_{0,i} \cdot Q_{k,i}
\]

With,

- \( \gamma_G \): partial factor for permanent actions
- \( G_k \): characteristic value of permanent action (e.g. dead load)
- \( \gamma_Q \): partial factor for variable actions
- \( Q_k \): characteristic value of variable action (e.g. live, snow and wind load)
- \( \psi_0 \): factor for combination value of a variable action

Serviceability limit state (SLS)

Characteristic combination:

\[
\sum_{j \geq 1} G_{k,j} + Q_{k.1} + \sum_{i > 1} \psi_{0,i} \cdot Q_{k,i}
\]

Frequent combination:

\[
\sum_{j \geq 1} G_{k,j} + \psi_{1.1} \cdot Q_{k.1} + \sum_{i > 1} \psi_{2,i} \cdot Q_{k,i}
\]

Quasi-permanent combination:

\[
\sum_{j \geq 1} G_{k,j} + \sum_{i \geq 1} \psi_{2,i} \cdot Q_{k,i}
\]

With,

- \( \psi_1 \): factor for frequent value of a variable action
- \( \psi_2 \): factor for quasi-permanent value of a variable action
Accidental limit state (ALS)

\[ \sum_{j \geq 1} G_{k,j} + A_d + (\psi_{1,1} \text{ or } \psi_{2,1}) \cdot Q_{k,1} + \sum_{i > 1} \psi_{2,i} \cdot Q_{k,i} \]

Seismic combination:

\[ \sum_{j \geq 1} G_{k,j} + A_{Ed} + \sum_{i \geq 1} \psi_{2,i} \cdot Q_{k,i} \]

With,

- \( A_d \) design value of an accidental action
- \( A_{Ed} \) design value of seismic action

Full in-depth article about load combinations: [here](#)
Bending verification

Design bending stresses:

\[
\sigma_{m.y.d} = \frac{M_{y.d}}{I_y} \cdot \frac{h}{2}
\]
\[
\sigma_{m.z.d} = \frac{M_{z.d}}{I_z} \cdot \frac{w}{2}
\]

With,

- \(M_d\) design bending moment around y/z-axis
- \(I\) moment of inertia around y/z-axis
- \(h\) cross-sectional height
- \(w\) cross-sectional width

Design bending strengths (EN 1995-1-1 (2.17)):

\[
f_{m.y.d} = k_{mod} \cdot \frac{f_{m.y.k}}{\gamma_m}
\]
\[
f_{m.z.d} = k_{mod} \cdot \frac{f_{m.z.k}}{\gamma_m}
\]

With,

- \(k_{mod}\) modification factor (EN 1995-1-1 Table 3.1)
- \(f_{m.k}\) characteristic bending strength of timber material
- \(\gamma_m\) partial factor (EN 1995-1-1 Table 2.3)

Bending verification (EN 1995-1-1 (6.11) + (6.12))

\[
k_m = 0.7/1.0
\]
\[
\frac{\sigma_{m.y.d}}{f_{m.y.d}} + k_m \cdot \frac{\sigma_{m.z.d}}{f_{m.z.d}} \leq 1
\]
\[
k_m \cdot \frac{\sigma_{m.y.d}}{f_{m.y.d}} + \frac{\sigma_{m.z.d}}{f_{m.z.d}} \leq 1
\]

EN 1995-1-1 6.1.6 (2)
Shear verification

Design shear stress:

\[ \tau_{v,d} = \frac{3}{2} \cdot \frac{V_d}{w \cdot h} \]

With,

- \( V_d \): design shear force
- \( h \): cross-sectional height
- \( w \): cross-sectional width

Design shear strengths (EN 1995-1-1 (2.17)):

\[ f_{v,d} = k_{mod} \frac{f_{v,k}}{\gamma_m} \]

With,

- \( k_{mod} \): modification factor (EN 1995-1-1 Table 3.1)
- \( f_{v,k} \): characteristic shear strength of timber material
- \( \gamma_m \): partial factor (EN 1995-1-1 Table 2.3)

Shear verification (EN 1995-1-1 (6.13))

\[ \frac{\tau_{v,d}}{f_{v,d}} \leq 1 \]
**Compression verification (columns)**

Design compression stress:

\[
\sigma_{c.0.d} = \frac{N_d}{w \cdot h}
\]

With,

- \(N_d\): design normal force
- \(h\): cross-sectional height
- \(w\): cross-sectional width

Design compression strength – parallel to grain (EN 1995-1-1 (2.17)):

\[
f_{c.0.d} = k_{mod} \frac{f_{c.0.k}}{\gamma_m}
\]

With,

- \(k_{mod}\): modification factor (EN 1995-1-1 Table 3.1)
- \(f_{c.0.k}\): characteristic compression strength parallel to grain
- \(\gamma_m\): partial factor (EN 1995-1-1 Table 2.3)

Compression verification (EN 1995-1-1 (6.2))

\[
\frac{\sigma_{c.0.d}}{f_{c.0.d}} \leq 1
\]
Buckling verification (columns)

Buckling length: \( l \)

Radius of inertia: \( i = \sqrt{\frac{l}{w \cdot h}} \)

Slenderness ratio: \( \lambda = \frac{l}{i} \)

Relative slenderness ratio (EN 1995-1-1 (6.21)): \( \lambda_{rel.y} = \frac{\lambda}{\pi} \cdot \frac{f_{c.0.k}}{E_{0.g.05}} \)

\( \beta_c \) factor (EN 1995-1-1 (6.29)) \( \beta_c \)

Instability factor (EN 1995-1-1 (6.27)) \( k = 0.5 \cdot (1 + \beta_c \cdot (\lambda_{rel.y} - 0.3) + \lambda_{rel.y}^2) \)

Buckling reduction factor coefficient (EN 1995-1-1 (6.25)) \( k_c = \frac{1}{k + \sqrt{k^2 - \lambda_{rel.y}^2}} \)

Utilization check one axial bending (EN 1995-1-1 (6.23)) \( \frac{\sigma_{c.0.d}}{k_c \cdot f_{c.0.d}} + \frac{\sigma_{m.d}}{f_{m.d}} \leq 1 \)

With,

\( f_{c.0.k} \) characteristic compression strength parallel to grain
\( E_{0.g.05} \) E-modulus parallel to grain, 5% fractile
\( l \) moment of inertia
\( h \) cross-sectional height
\( w \) cross-sectional width
Material properties

E-modulus: \[ E = 0.21 \cdot 10^6 \text{MPa} \]

Density: \[ \rho = 7850 \frac{\text{kg}}{\text{m}^3} \]

Poissons ratio: \[ \nu = 0.3 \]

Shear modulus: \[ G = \frac{E}{2 \cdot (1 + \nu)} \]

Yield and ultimate strength: \[ f_y, f_u \]

Check out the tables on Eurocodeapplied.com (link)

Strength properties of bolts (EN 1993-1-8 Table 3.1)

<table>
<thead>
<tr>
<th>Property</th>
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<td>Yield strength ( f_{yb} ) (MPa)</td>
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<td>Ultimate tensile strength ( f_{ub} ) (MPa)</td>
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</tr>
</tbody>
</table>

Ultimate tensile strength of welds:

\[ f_u = f_u \] of the weaker steel plate

Correlation factor

(EN 1993-1-8 Table 4.1)
**Geometric properties**

Cross-sectional height: \( h \)

Cross-sectional width: \( w \)

Web thickness: \( t_w \)

Flange thickness: \( t_f \)

Root radius: \( r \)

You can find these and many more parameters on Eurocodeapplied.com ([link](#))

Bolt nominal diameter: \( d \)

Bolt Hole diameter: \( d_0 \)

Stress area (threaded part of bolt): \( A_s \)

You can find these and many more parameters on Eurocodeapplied.com ([link](#))

Weld throat thickness: \( a \)
**ULS compression verification**

Compressive design resistance (EN 1993-1-1 (6.10)):

\[ N_{c,Rd} = \frac{A \cdot f_y}{\gamma_{M0}} \]

With,

- \( A \): full/effective cross-sectional area depending on the cross-section class
- \( f_y \): steel yield strength
- \( \gamma_{M0} \): partial factor

Compression verification (EN 1993-1-1 (6.9)):

\[ \frac{N_{Ed}}{N_{c,Rd}} \leq 1.0 \]

With,

- \( N_{Ed} \): design compression force (normal force)

**ULS bending verification**

Bending design resistance (EN 1993-1-1 (6.13)):

\[ M_{c,Rd} = \frac{W_{pl} \cdot f_y}{\gamma_{M0}} \]

With,

- \( W_{pl} \): plastic section modulus (cross-section classes 1+2). You can find the value here.

Bending verification (EN 1993-1-1 (6.12)):

\[ \frac{M_{Ed}}{M_{c,Rd}} \leq 1.0 \]

With,

- \( M_{Ed} \): design bending moment
**ULS shear verification**

Shear design resistance (EN 1993-1-1 (6.18)):

$$V_{pl,Rd} = A_v \frac{f_y}{\sqrt{3} \gamma_{M0}}$$

With,

- $A_v$: shear area. You can find the values [here](#).

Check if shear buckling verification is required (EN 1993-1-1 (6.22)):

$$\varepsilon \leq \sqrt{\frac{235\text{MPa}}{f_y}}$$

$$\eta \leq 1.0$$

$$\frac{h_w}{t_w} \leq 72 \cdot \frac{\varepsilon}{\eta}$$

Shear verification (EN 1993-1-1 (6.17)):

$$\frac{V_{Ed}}{V_{c,Rd}} \leq 1.0$$

With,

- $V_{Ed}$: design shear force
ULS flexural buckling verification (columns)

Buckling length: \( l \)

Radius of inertia:
\[
i = \sqrt{\frac{I}{w \cdot h}}
\]

Slenderness (EN 1993-1-1 6.3.1.3 (1)):
\[
\lambda_1 = \pi \frac{E}{f_y}
\]

Non-dimensional slenderness:
\[
\lambda = \frac{l}{i} \frac{1}{\lambda_1}
\]

Buckling curve (EN 1993-1-1 Table 6.1):
\[ \alpha \]

Reduction factors (EN 1993-1-1 (6.49)):
\[
\phi = 0.5 \cdot (1 + \alpha \cdot (\lambda - 0.2) + \lambda^2)
\]

\[
\chi = \frac{1}{\phi + \sqrt{\phi^2 - \lambda^2}}
\]

Design buckling resistance (EN 1993-1-1 (6.47))
\[
N_{b,Rd} = \frac{\chi \cdot A \cdot f_y}{\gamma_{M1}}
\]

With,

A full/effective cross-sectional area depending on the cross-section class

Utilization check (EN 1993-1-1 (6.46))
\[
\frac{N_{Ed}}{N_{b,Rd}} \leq 1
\]
Fillet weld design – directional method

Normal stress perpendicular to throat:

\[
\sigma_{90} = \frac{N}{2 \cdot l_{eff} \cdot a \cdot \sqrt{2}} + \frac{M}{2 \cdot l_{eff}^2 \cdot \frac{a}{6} \cdot \sqrt{2}}
\]

Shear stress parallel to the axis of weld:

\[
\tau_0 = \frac{V}{2 \cdot l_{eff} \cdot a}
\]

Shear stress perpendicular to the axis of weld:

\[
\tau_{90} = \sigma_{90}
\]

Verification criteria 1 (EN 1993-1-8 (4.1)):

\[
\sqrt{\sigma_{90}^2 + 3(\tau_0^2 + \tau_{90}^2)} \leq \frac{f_u}{\beta_w \cdot \gamma_{M2}}
\]

Verification criteria 2 (EN 1993-1-8 (4.1)):

\[
\sigma_{90} \leq 0.9 \frac{f_u}{\gamma_{M2}}
\]
**Fillet weld design – simplified method**

Normal stress in weld:

\[ \sigma_N = \frac{N}{l_w \cdot a} + \frac{M}{W} \]

With,

- \( l_w \) length of weld
- \( W \) section modulus of weld

Shear stress in weld:

Shear stress perpendicular to the axis of weld:

\[ \tau_v = \frac{V_d}{l_w \cdot a} \]

Resulting stress:

\[ F_{w,Ed} = \sqrt{\sigma_N^2 + \tau_v^2} \]

Resistance stress:

\[ F_{w,Rd} = \frac{f_u}{\sqrt{3} \cdot \beta_w \cdot \gamma_{M2}} \]

Utilization check (EN 1993-1-8 (4.2)):

\[ F_{w,Ed} \leq F_{w,Rd} \]

Full in-depth article about fillet weld design [here](#).
RC DESIGN

Material properties

E-modulus concrete \( E \)
(dependent on concrete strength class):

Partial factor – concrete: \( \gamma_c \)

Partial factor – reinforcement: \( \gamma_s \)

Characteristic cylinder compressive strength: \( f_{ck} \)

Design compressive strength: \( f_{cd} = \frac{f_{ck}}{\gamma_c} \)

Mean tensile concrete strength: \( f_{ctm} \)

Design tensile strength: \( f_{ctd} = \frac{f_{ctm}}{\gamma_c} \)

Yield strength of reinforcement: \( f_{yk} \)

Design yield strength: \( f_{yd} = \frac{f_{yk}}{\gamma_s} \)

You can find the values of these properties on Eurocodeapplied.com (link)
**ULS Bending verification**

Lever arm of longitudinal reinforcement to compression fibre

\[ \mu = \frac{M_d}{w \cdot d^2 \cdot \eta \cdot f_{cd}} \]

\[ \omega = 1 - \sqrt{1 - 2 \cdot \mu} \]

Required longitudinal reinforcement:

\[ A_{s,req} = \omega \cdot \frac{w \cdot d \cdot \eta \cdot f_{cd}}{f_{yd}} \]

With,

- \( M_d \) bending moment
- \( w \) width of RC beam
- \( \eta \) factor dependent on concrete class

Degree of reinforcement checks:

\[ \omega_{min} = \max(0.26 \frac{f_{ctm}}{f_{yk}} \cdot \frac{f_{yd}}{f_{cd}} ; 0.0013 \frac{f_{yd}}{f_{cd}}) \]

\[ \omega_{bal} = \lambda \frac{\varepsilon_{cu3}}{\varepsilon_{cu3} \cdot \varepsilon_{yd}} \]

\[ \omega_{max} = 0.044 \frac{f_{yd}}{\eta \cdot f_{cd}} \]

Check 1:

\[ \omega > \omega_{min} \]

Check 2:

\[ \omega < \omega_{bal} \]
Check 3:

Minimum reinforcement (EN 1992-1-1 9.2.1.1 (9.1N)):

\[ A_{s,\text{min}} = \max \left\{ 0.26 \frac{f_{ctm}}{f_{yk}} \cdot w \cdot d, 0.0013 \cdot w \cdot d \right\} \]

\[ \omega < \omega_{\text{max}} \]

**ULS Shear verification**

First, check if shear reinforcement is required.

**Members not requiring design shear reinforcement**

EN 1992-1-1 6.2.2 (1):

\[ k = 1 + \frac{200}{\sqrt{d \text{ mm}}} \]

\[ \rho_1 = \min\left(\frac{A_s}{w \cdot d}; 0.02\right) \]

Design value of shear resistance (EN 1992-1-1 (6.2.a)):

\[ V_{Rd,c} = \max \left\{ \frac{0.18}{\gamma_c} \cdot k \cdot (100 \rho_1 \cdot f_{ck})^{\frac{1}{3}}, \frac{0.051}{\gamma_c} \cdot k^{\frac{3}{2}} \cdot \sqrt{f_{ck}} \right\} \]

\[ V_{Rd,c} = V_{Rd,c} \cdot w \cdot h \]

Shear reinforcement required if:

\[ V_{Ed} > V_{Rd,c} \]

With,

- \( h \): height of RC beam
- \( V_{Ed} \): width of RC beam
Members requiring design shear reinforcement

EN 1992-1-1 Figure 6.5
\[ z = 0.9 \cdot d \]

Coefficient taking into account state of stress in compression chord:
\[ \alpha_{cw} \]

Strength reduction factor for concrete cracked in shear (EN 1992-1-1 (6.9)):
\[ \nu_1 \]

Angle between concrete compression strut and beam axis perp. to shear force:
\[ \theta \]

Shear resistance (EN 1992-1-1 (6.9)):
\[ V_{Rd,max} = \alpha_{cw} \cdot w \cdot z \cdot \nu_1 \cdot \frac{f_{cd}}{\cot(\theta) + \tan(\theta)} \]

Verification
\[ \eta = \frac{V_{Ed}}{V_{Rd,max}} \]

Reduction of design yield strength of reinforcement (EN 1992-1-1 (6.8)):
\[ f_{ywd} = 0.8 \cdot f_{yk} \]

Shear links (EN 1992-1-1 (6.8)):
\[ A_{sw} = \frac{V_{Ed}}{z \cdot f_{ywd} \cdot \cot(\theta)} \]

Inclination of shear reinforcement:
\[ \alpha \]

Max. spacing (EN 1992-1-1 (9.6N)):
\[ s_{l,max} = 0.75 \cdot d \cdot (1 + \cot(\alpha)) \]
SLS Crack width verification

Crack width limit (EN 1992-1-1 Table 7.1N)

Final creep coefficient \( \phi \)

E-modulus of concrete long-term (quasi-permanent):

\[
E_{c,eff} = \frac{E_{cm}}{1 + \phi}
\]

Long-term steel-concrete ratio:

\[
\alpha_s = \frac{E_s}{E_{c,eff}}
\]

Equilibrium of 1. moment of area:

\[
w \cdot x \cdot \frac{x}{2} = \alpha_s \cdot A_s \cdot (d - x)
\]

Neutral axis (solving for \( x \)):

\[
x = \frac{\alpha_s \cdot A_s}{w} \cdot (-1 + \sqrt{1 + \frac{2 \cdot w \cdot d}{\alpha_s \cdot A_s}})
\]
**RC DESIGN**

Stress in reinforcement: \[ \sigma_s = \frac{M_{qp}}{(d - \frac{x}{3}) \cdot A_s} \]

Factor dependent on duration of load (EN 1992-1-1) \( k_t \)

\[ f_{ct.eff} = f_{ctm} \]

Effective height (EN 1992-1-1 (7.3.2 (3))):

\[ h_{c.eff} = \min(2.5 \cdot (h - d); \frac{h - x}{3}; \frac{h}{2}) \]

\[ \rho_{p.eff} = \frac{A_s}{h_{c.eff} \cdot w} \]

EN 1992-1-1 (7.9):

\[ \varepsilon_{sm} - \varepsilon_{cm} = \max \left\{ \frac{\sigma_s - k_t \cdot f_{ct.eff} \cdot (1 + \alpha_s \cdot \rho_{p.eff})}{E_s}, \frac{0.6 \cdot \sigma_s}{E_s} \right\} \]

Coefficient (EN 1992-1-1 (7.11)) \( k_1 \)

Coefficient (EN 1992-1-1 (7.11)) \( k_2 \)

Max. cracking spacing: \( s_{r.max} = 3.4 \cdot c + 0.425 \cdot k_1 \cdot k_2 \cdot \frac{d_s}{\rho_{p.eff}} \)

Verification: \( s_{r.max} < 5 \cdot (c + \frac{d_s}{2}) \)
Crack width:  
\[ w_k = s_{r,\text{max}} \cdot (\varepsilon_{sm} - \varepsilon_{cm}) \]

Utilization:  
\[ \eta = \frac{w_k}{w_{\text{max}}} \]

**SLS Deflection verification**

The deflection calculation of reinforced concrete elements is not as straightforward as for timber or steel structures. However, according to Eurocode EN 1992-1-1 the deflection requirement is likely to be satisfied if the span – effective depth ratio is less than the values given in EN 1992-1-1 Table 7.4N.

Click [here](https://example.com) for a reinforced concrete beam design guide.